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2007 J. Phys. A: Math. Theor. 40 7043

(http://iopscience.iop.org/1751-8121/40/25/S59)

IOP Publishing | Journal of Physics A: Mathematical and Theoretical

J. Phys. A: Math. Theor. **40** (2007) 7043–7048 doi:10.1088/1751-8113/40/25/S59

Cosmological landscape and Euclidean quantum gravity

A O Barvinsky[1,2] **and A Yu Kamenshchik**[3,4]

[1] Theory Department, Lebedev Physics Institute, Leninsky Prospect 53, 119991 Moscow, Russia
[2] Sektion Physik, LMU, Theresienstr 37, Munich, Germany
[3] Dipartimento di Fisica and INFN, Via Irnerio 46, 40126 Bologna, Italy
[4] L D Landau Institute for Theoretical Physics, Kosygin Str. 2, 119334 Moscow, Russia

E-mail: barvin@lpi.ru and kamenshchik@bo.infn.it

Received 30 October 2006, in final form 11 January 2007
Published 6 June 2007
Online at stacks.iop.org/JPhysA/40/7043

Abstract

Quantum creation of the universe is described by the *density matrix* defined by the Euclidean path integral. This yields an ensemble of universes—a cosmological landscape—in a mixed quasi-thermal state which is shown to be dynamically more preferable than the pure quantum state of the Hartle–Hawking type. The latter is suppressed by the infinitely large positive action of its instanton, generated by the conformal anomaly of quantum matter. The Hartle–Hawking instantons can be regarded as posing initial conditions for Starobinsky solutions of the anomaly driven de Sitter expansion, which are thus dynamically eliminated by infrared effects of quantum gravity. The resulting landscape of hot universes treated within the cosmological bootstrap (the self-consistent back reaction of quantum matter) turns out to be limited to a bounded range of the cosmological constant, which rules out a well-known infrared catastrophe of the vanishing cosmological constant and suggests an ultimate solution to the problem of unboundedness of the cosmological action in Euclidean quantum gravity.

PACS numbers: 04.60.Gw, 04.62.+v, 98.80.Bp, 98.80.Qc

1. Introduction

The ideas of quantum cosmology [1, 2] and Euclidean quantum gravity [3, 4] are again attracting attention. One of the reasons is the fact that the landscape of string vacua is too big [5] to hope that a reasonable selection mechanism can be successfully worked out within string theory itself. Thus, it is expected that other methods have to be invoked, at least some of them appealing to the construction of the cosmological wavefunction [6–9]. This quantum state arises as a result of quantum tunnelling from the classically

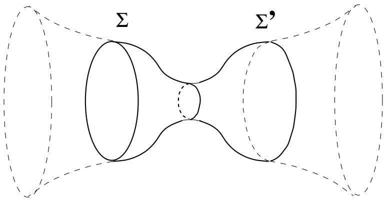

Figure 1. Picture of instanton representing the density matrix. Dashed lines depict the Lorentzian universe nucleating from the instanton at the minimal surfaces Σ and Σ'.

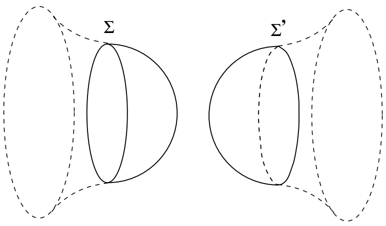

Figure 2. Density matrix of the pure Hartle–Hawking state represented by the union of two vacuum instantons.

forbidden state of the gravitational field. The Hartle–Hawking wavefunction of the universe [3, 4] describes nucleation of the de Sitter universe from the Euclidean four-dimensional hemisphere, $\Psi_{\mathrm{HH}} \sim \exp(-S_{\mathrm{E}}) = \exp(3\pi/2G\Lambda)$ and has a negative action which diverges to $-\infty$ for the cosmological constant $\Lambda \to 0$. This implies a well-known infrared catastrophe of small cosmological constant—a vanishing Λ is infinitely more probable than any positive one. Alternative tunnelling proposals for the wavefunction of the universe in the form of Linde [10] or Vilenkin [11] give preference to big values of Λ, which opens the possibility for conclusions opposite to the Hartle–Hawking case. In particular, the inclusion of one-loop effects allows one to shift most probable values of the effective cosmological constant from zero to a narrow highly peaked range compatible with the requirements of inflation [12].

In this work we study the Hartle–Hawking prescription of the Euclidean path integration taking into account essentially *nonlocal* quantum effects mediated by nonlocal terms of non-vacuum nature. The core of our suggestion is a simple observation that the presence of radiation implies a statistical ensemble described by the density matrix, rather than a pure state assumed in [7, 8]. Density matrix in Euclidean quantum gravity [13], $\rho[\varphi, \varphi']$, originates from an instanton with two disjoint boundaries Σ and Σ' associated respectively with its two entries, see figure 1. Note that mixed nature of the quantum state is fundamental and the origin of impurity is not caused by coarse graining or tracing out environmental degrees of freedom.

In contrast, the pure density matrix of the Hartle–Hawking state corresponds to the situation when the instanton bridge between Σ and Σ' is broken, so that topologically the instanton is a union of two disjoint hemispheres. Each of the half-instantons smoothly closes up at its pole which is a regular internal point of the Euclidean spacetime ball, see figure 2—a picture illustrating the factorization of $\hat{\rho} = |\Psi_{\mathrm{HH}}\rangle\langle\Psi_{\mathrm{HH}}|$.

When calculated in the saddle-point approximation the density matrix automatically gives rise to radiation whose thermal fluctuations destroy the Hartle–Hawking instanton. Namely, the radiation stress tensor prevents the half-instantons of the above type from closing and, thus, forces the tubular structure on the full instanton supporting the thermodynamical nature of the physical state. The existence of radiation, in its turn, naturally follows from the partition function of this state. The partition function originates from integrating out the field φ in the coincidence limit $\varphi' = \varphi$. (This procedure is equivalent to taking the trace of the operator $\exp(-\beta H)$ inherent in the calculation of the partition function for a system with the Hamiltonian H.) This corresponds to the identification of Σ' and Σ, so that the underlying instanton acquires toroidal topology. Its points are labelled by the periodically identified Euclidean time, a period being related to the inverse temperature of the quasi-equilibrium radiation. The back reaction of this radiation supports the instanton geometry in which this radiation exists, and we derive the equation which makes this bootstrap consistent.

We show that for the radiation of conformally invariant fields the analytical and numerical solution of the bootstrap equations yields a set of instantons—a landscape—only in the bounded range of Λ,

$$\Lambda_{\min} < \Lambda < \Lambda_{\max}. \tag{1}$$

This set consists of the countable sequence of one-parameter families of instantons which we call garlands, labelled by the number $k = 1, 2, 3, \ldots$ of their elementary links. Each of these families spans a continuous subset, $\Lambda_{\min}^{(k)} < \Lambda \leqslant \Lambda_{\max}^{(k)}$, belonging to (1). These subsets of monotonically decreasing lengths $\Lambda_{\max}^{(k)} - \Lambda_{\min}^{(k)} \sim 1/k^4$ do not overlap and their sequence has an accumulation point at the upper boundary $\Lambda_{\max}$ of the range (1). Each of the instanton families at its upper boundary $\Lambda_{\max}^{(k)}$ saturates with the static Einstein universe filled by a hot equilibrium radiation with the temperature $T_{(k)} \sim m_P/\ln k^2$, $k \gg 1$, and having the *negative* decreasing with k action $\Gamma_0^{(k)} \sim -\ln^3 k^2/k^2$. Remarkably, all values of Λ below the range (1), $\Lambda < \Lambda_{\min}$, are completely ruled out either because of the absence of instanton solutions or because of their *infinitely large positive* action.

The details of our approach have recently been presented in [14], while here we discuss its basic points and, in particular, establish its relation to the well-known Starobinsky model [15] of the self-consistent de Sitter expansion driven by the conformal anomaly of quantum fields.

2. The effective action, generalized Friedmann equations and bootstrap

As shown in [14] the effective action of the cosmological model with a generic spatially closed FRW metric $\mathrm{d}s^2 = N^2(\tau)\,\mathrm{d}\tau^2 + a^2(\tau)\,\mathrm{d}^2\Omega^{(3)} = a^2(\mathrm{d}\eta^2 + \mathrm{d}^2\Omega^{(3)})$ sourced by *conformal* quantum matter has the form

$$\Gamma[a(\tau), N(\tau)] = 2\int_{\tau_-}^{\tau_+} \mathrm{d}\tau \left(-\frac{a\dot{a}^2}{N} - Na + NH^2a^3\right) + 2B\int_{\tau_-}^{\tau_+} \mathrm{d}\tau \left(\frac{\dot{a}^2}{Na} - \frac{1}{6}\frac{\dot{a}^4}{N^3a}\right)$$
$$+ B\int_{\tau_-}^{\tau_+} \mathrm{d}\tau\, N/a + F\left(2\int_{\tau_-}^{\tau_+} \mathrm{d}\tau\, N/a\right), \tag{2}$$

where $a(\tau)$ is the cosmological radius, $N(\tau)$ is the lapse function, $H^2 = \Lambda/3$ and integration runs between two turning points at $\tau_\pm$, $\dot{a}(\tau_\pm) = 0$. Here the first line is the classical part, the second line is the contribution of the conformal transformation to the metric of the static instanton $\mathrm{d}\bar{s}^2 = \mathrm{d}\eta^2 + \mathrm{d}^2\Omega^{(3)}$ (η is the conformal time) and the last line is the one-loop action on this static instanton. The conformal contribution $\Gamma_{1-\mathrm{loop}}[g] - \Gamma_{1-\mathrm{loop}}[\bar{g}]$ is determined by the coefficients of $\Box R$, the Gauss–Bonnet invariant $E = R_{\mu\nu\alpha\gamma}^2 - 4R_{\mu\nu}^2 + R^2$ and Weyl tensor term in the conformal anomaly $g_{\mu\nu}\delta\Gamma_{1-\mathrm{loop}}/\delta g_{\mu\nu} = g^{1/2}\big(\alpha\Box R + \beta E + \gamma C_{\mu\nu\alpha\beta}^2\big)/4(4\pi)^2$. Specifically this contribution can be obtained by the technique of [16]; it contains higher derivative terms $\sim\ddot{a}^2$ which produce ghost instabilities in solutions of effective equations. However, such terms are proportional to the coefficient α which can be put to zero by adding

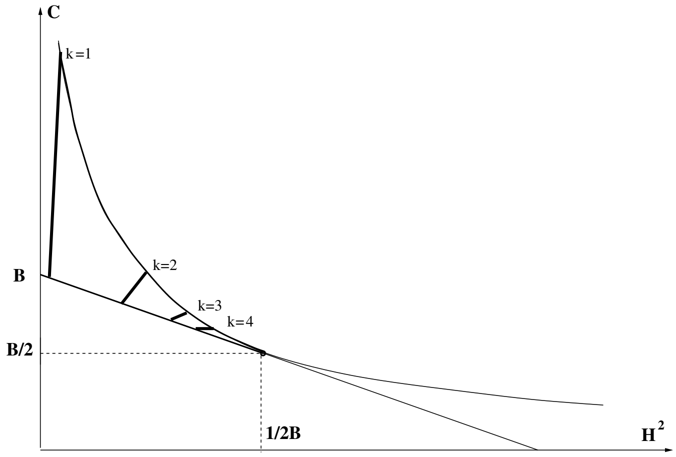

Figure 3. Instanton domain in the (H^2, C) plane. Garland families are shown for $k = 1, 2, 3, 4$. Their sequence accumulates at the critical point $(1/2B, B/2)$.

the following finite *local* counterterm:

$$\Gamma_R[g] = \Gamma_{1-\text{loop}}[g] + \frac{1}{2(4\pi)^2}\frac{\alpha}{12}\int d^4x\, g^{1/2}R^2(g). \tag{3}$$

This ghost-avoidance renormalization is justified by the requirement of consistency of the theory at the quantum level. The contribution $\Gamma_R[g] - \Gamma_R[\bar{g}]$ to the *renormalized* action then gives the second line of (2) with $B = 3\beta/4$.

The static instanton with a period η_0 playing the role of inverse temperature contributes $\Gamma_{1-\text{loop}}[\bar{g}] = E_0\eta_0 + F(\eta_0)$, where the vacuum energy E_0 and free energy $F(\eta_0)$ are the typical boson and fermion sums over field oscillators with energies ω on a unit 3-sphere $E_0 = \pm\sum_\omega \omega/2$, $F(\eta_0) = \pm\sum_\omega \ln(1 \mp e^{-\omega\eta_0})$. The renormalization (3) which should be applied also to $\Gamma_{1-\text{loop}}[\bar{g}]$ modifies E_0, so that $\Gamma_R[\bar{g}] = C_0\eta_0 + F(\eta_0)$, $C_0 \equiv E_0 + 3\alpha/16$. This gives the third line of equation (2) with $C_0 = B/2$. This universal relation between C_0 and $B = 3\beta/4$ follows from the known anomaly coefficients [17] and the Casimir energy in a static universe [18] for scalar, Weyl spinor and vector fields.

The Euclidean Friedmann equation looks now as

$$\frac{\dot{a}^2}{a^2} + B\left(\frac{1}{2}\frac{\dot{a}^4}{a^4} - \frac{\dot{a}^2}{a^4}\right) = \frac{1}{a^2} - H^2 - \frac{C}{a^4}, \tag{4}$$

$$C = B/2 + F'(\eta_0), \qquad \eta_0 = 2\int_{\tau_-}^{\tau_+} d\tau/a(\tau). \tag{5}$$

The contribution of the nonlocal $F(\eta_0)$ in (2) reduces here to the radiation *constant* C as a *nonlocal functional* of $a(\tau)$, determined by the *bootstrap* equation (5), $F'(\eta_0) \equiv dF(\eta_0)/d\eta_0 > 0$ being the energy of a hot gas of particles, which adds to their vacuum energy $B/2$.

Periodic instanton solutions of equations (4)–(5) exist only inside the curvilinear wedge of (H^2, C) plane between bold segments of the upper hyperbolic boundary and the lower straight line boundary of figure 3,

$$4CH^2 \leqslant 1, \qquad C \geqslant B - B^2H^2, \qquad BH^2 \leqslant 1/2. \tag{6}$$

Below this domain the solutions are either complex and aperiodic or suppressed by *infinite positive* Euclidean action. Indeed, a smooth Hartle–Hawking instanton with $a_- = 0$ yields

$\eta_0 \to \infty$ in view of (5), so that $F(\eta_0) \sim F'(\eta_0) \to 0$. Therefore, its on-shell action

$$\Gamma_0 = F(\eta_0) - \eta_0 F'(\eta_0) + 4\int_{a_-}^{a_+} \frac{\mathrm{d}a\dot{a}}{a}\left(B - a^2 - \frac{B\dot{a}^2}{3}\right) \tag{7}$$

due to $B > 0$ diverges to $+\infty$ at $a_- = 0$ and completely rules out pure-state instantons [14]. For the instanton garlands, obtained by gluing together into a torus k copies of a simple instanton [19], the formalism is the same as above except that the conformal time in (5) and the integral term of (7) should be multiplied by k. As in the case of $k = 1$, garland families interpolate between the lower and upper boundaries of (6). They exist for all k, $1 \leqslant k \leqslant \infty$, and their infinite sequence accumulates at the critical point $C = B/2$, $H^2 = 1/2B$, with the on-shell action $\Gamma_0^{(k)} \simeq -B\ln^3 k^2/4k^2\pi^2$ which, in contrast to the tree-level garlands of [19], is not additive in k.

3. Conclusions: Euclidean quantum gravity and the Starobinsky model

Elimination of the Hartle–Hawking vacuum instantons implies also ruling out well-known solutions in the Starobinsky model of the anomaly (and vacuum energy) driven de Sitter expansion [15]. Such solutions (generalized to the case of a nonzero 'bare' cosmological constant $3H^2$) can be obtained by the usual Wick rotation from the solutions of (4) with the thermal contribution switched off. The corresponding Euclidean Friedmann equation reads

$$\frac{\dot{a}^2}{a^2} - \frac{1}{a^2} + \frac{B}{2}\left(\frac{\dot{a}^2}{a^2} - \frac{1}{a^2}\right)^2 + H^2 = 0, \tag{8}$$

and has a generic solution of the form $a = \sin h\tau/h$, $1/a^2 - \dot{a}^2/a^2 = h^2$, with the following two values of the Hubble parameter $h = h_\pm$, $h_\pm^2 = (1/B)(1 \pm \sqrt{1 - 2BH^2})$. For $H = 0$, this is exactly the Euclidean version of Starobinsky's solution [15] with $h_+^2 = 2/B$. For larger $H^2 < 1/2B$, we have two families of exactly de Sitter instantons which can be regarded as initial conditions for the 'generalized' solutions of Starobinsky. However, all of them are ruled out by their infinite positive Euclidean action.

What remains is a quasi-thermal ensemble of non-vacuum models in the bounded cosmological constant range (1) with $\Lambda_{\min} > 0$ and $\Lambda_{\max} = 3/2B$ and with a finite value of their effective Euclidean action. This implies the elimination of the infrared catastrophe of $\Lambda \to 0$ and the ultimate solution to the problem of unboundedness of the *on-shell* cosmological action in Euclidean quantum gravity. As a byproduct, this also suggests strong constraints on the cosmological constant apparently applicable to the resolution of the cosmological constant problem.

Acknowledgments

AK is grateful to A A Starobinsky and A Vilenkin for useful discussions. AB is indebted to C Deffayet for stimulating discussions and the hospitality of IAP in Paris. AB was supported by the RFBR grant 05-02-17661 and the LSS 4401.2006.2 and SFB 375 grants. AK was partially supported by the RFBR grant 05-02-17450 and the grant LSS 1757.2006.2.

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